MATH 1A - MOCK MIDTERM 2

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Name: ____________________

Instructions: This is a mock midterm, designed to give you an idea of what the actual midterm will look like. Make sure you do it, the actual exam will be very similar to this one (in length and in difficulty)!

1		15
2		15
3		**50**
4		20
Bonus 1		5
Bonus 2		5
Total		100

Date: Friday, July 15th, 2011.

1. *(15 points)* Using **the definition** of the derivative, find $f'(1)$, where $f(x) = \frac{1}{x}$.

2. *(15 points)* Using **the definition** of the derivative, calculate the derivative of $f(x) = \sqrt{x} + x$

3. *(50 points, 5 points each)* Find the derivatives of the following functions:
   (a) $f(x) = \frac{x+e^x}{e^x+1}$ (no need to simplify)

   (b) $f(x) = -\tan^{-1}(\frac{1}{x})$ (simplify your answer! Do you notice anything? See solution for an interesting comment!)

   (c) $f''(x)$, where $f(x) = \sin(x)e^x$

   (d) The equation of the tangent line to $y = \frac{e^x}{x}$ at the point $(1, e)$

(e) $f(x) = \ln(\sqrt{x^2+1})$ (simplify your answer!)

(f) $f(x) = \ln(\ln(\ln(x)))$

(g) y' where $x^2 + xy + y^2 = 3$

(h) $f(x) = x^{\cos(x)}$

(i) y' at $(0, -2)$, where $y^2(y^2 - 4) = x^2(x^2 - 5)$

(j) y', where $x^y = y^x$ (**Hint:** Take $\ln$'s)

4. *(20 points)* Remember that one of the following problems will be for sure on your exam: (those are problems 40, 41, 42 in section 3.5)

Problem 1: Show that the equation of the tangent line to the ellipse:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

at the point (x_0, y_0) is

$$\frac{x_0 x}{a^2} + \frac{y_0 y}{b^2} = 1$$

Problem 2: Show that the equation of the tangent line to the hyperbola:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

at the point (x_0, y_0) is

$$\frac{x_0 x}{a^2} - \frac{y_0 y}{b^2} = 1$$

Problem 3: Show that the sum of the $x-$ and $y-$ intercepts of any tangent line to the curve $\sqrt{x} + \sqrt{y} = \sqrt{c}$ is equal to c.

Bonus 1 *(5 points)* Give examples of functions f and g with $f(x) \leq g(x)$, but:

(i) $f'(x) \leq g'(x)$
(ii) $f'(x) = g'(x)$
(iii) $f'(x) \geq g'(x)$

Hint: If you want to, one of your functions can be the zero function!

Note: This problem is meant to show you that derivatives can behave in very strange ways. We've seen that this is not the case with limits, i.e. if $f(x) \leq g(x)$, $\lim_{x\to a} f(x) \leq \lim_{x\to a} g(x)$, and we will also see that this is not the case with integrals, i.e. if $f(x) \leq g(x)$, then $\int_a^b f(x)dx \leq \int_a^b g(x)dx$. There is more excitement to come on the actual midterm :)

Bonus 2 *(5 points)*

(a) Let

$$f(x) = \begin{cases} x \sin\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

Does $f'(0)$ exist?

(b) What about

$$f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$